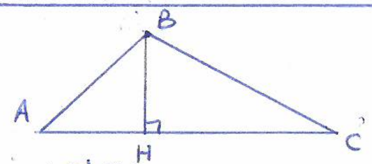
COURS : 1) $\vec{AB}\cdot\vec{AC} = AB \times AC \times \cos(\hat{A})$

$\vec{AB}\cdot\vec{AC} = xx' + yy'$

$\vec{AB}\cdot\vec{AC} = \vec{AH}\cdot\vec{AC}$

$\vec{AB}\cdot\vec{AC} = \frac{1}{2}\left(\|\vec{AB}+\vec{AC}\|^2 - \|\vec{AB}\|^2 - \|\vec{AC}\|^2\right)$

$\vec{AB}\cdot\vec{AC} = \frac{1}{2}\left(\|\vec{AB}\|^2 + \|\vec{AC}\|^2 - \|\vec{AB}-\vec{AC}\|^2\right)$



2) $\vec{AB}$ et $\vec{AC}$ sont orthogonaux $\Leftrightarrow$ $\vec{AB}\cdot\vec{AC} = 0$

3) A, B, C sont alignés $\Leftrightarrow$ $\exists k \in \mathbb{R}^* \,/\, \vec{AB} = k.\vec{AC}$

4) $(AB) \,//\, (CD) \Leftrightarrow \exists k \in \mathbb{R}^* \,/\, \vec{AB} = k.\vec{CD}$

Ex1 : 1) $\widehat{CBA} = \frac{\pi}{6}\,\text{rad} = 30°$ et $\sin(\widehat{CBA}) = \frac{AC}{BC}$ donc $\frac{4}{BC} = \sin(30°) = \frac{1}{2}$

donc $BC = 8\,\text{cm}$; d'où $AB^2 = BC^2 - AC^2 = 8^2 - 4^2 = 48$

donc $AB = \sqrt{48} = 4\sqrt{3}$ cm.

2) $\vec{BA}\cdot\vec{BC} = BA \times BC \times \cos(\hat{B}) = 4\sqrt{3} \times 8 \times \cos(30°) = 48$

$\vec{CA}\cdot\vec{CB} = CA \times CB \times \cos(\hat{C}) = 4 \times 8 \times \cos(60°) = 16$

$\vec{AB}\cdot\vec{AC} = 0$ car $\widehat{CAB} = 90°$

$\vec{CA}\cdot\vec{CD} = CA \times CD \times \cos(60°) = 4 \times 4 \times \cos(60°) = 8$

$\vec{AD}\cdot\vec{AB} = AD \times AB \times \cos(150°) = 4 \times 4\sqrt{3} \times \cos(150°) = -24$

$\vec{CB}\cdot\vec{CD} = CB \times CD \times \cos(120°) = 4 \times 8 \times \cos(120°) = -16$

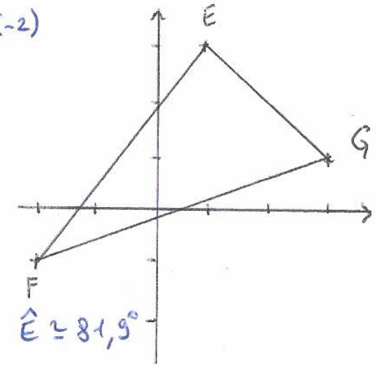
Ex2 : 1) $\vec{EF}\cdot\vec{EG} = \begin{pmatrix}-3\\-4\end{pmatrix}\cdot\begin{pmatrix}2\\-2\end{pmatrix} = -3\times 2 + (-4)\times(-2)$
$= 2$

$\cos(\hat{E}) = \frac{\vec{EF}\cdot\vec{EG}}{EF \times EG}$

or $EF = \sqrt{(-3)^2+(-4)^2} = 5$

$EG = \sqrt{2^2+(-2)^2} = 2\sqrt{2}$

donc $\cos(\hat{E}) = \frac{2}{10\sqrt{2}} = \frac{1}{5\sqrt{2}}$ donc $\hat{E} \simeq 81{,}9°$



2) $\vec{GE}\cdot\vec{GF} = \begin{pmatrix}-2\\2\end{pmatrix}\cdot\begin{pmatrix}-5\\-2\end{pmatrix} = (-2)\times(-5) + 2\times(-2) = 6$

$GE = \sqrt{(-2)^2+2^2} = 2\sqrt{2}$ et $GF = \sqrt{(-5)^2+(-2)^2} = \sqrt{29}$

donc $\cos(\hat{G}) = \frac{\vec{GE}\cdot\vec{GF}}{GE \times GF} = \frac{6}{2\sqrt{2}\times\sqrt{29}} = \frac{3}{\sqrt{58}}$ donc $\hat{G} \simeq 66{,}8°$

on déduit : $\hat{F} = 180° - \hat{E} - \hat{G} = 180° - 81{,}9° - 66{,}8° \simeq 31{,}3°$

Ex3 : 1) $\vec{AB}\cdot\vec{AC} = \begin{pmatrix}2-1\\m-(-2)\end{pmatrix}\cdot\begin{pmatrix}4-1\\2-m+2\end{pmatrix} = \begin{pmatrix}1\\m+2\end{pmatrix}\cdot\begin{pmatrix}3\\4-m\end{pmatrix}$

$= 1\times 3 + (m+2)(4-m) = -m^2 + 2m + 11$

2) ABC est rectangle en $\hat{A} \Leftrightarrow \hat{A} = 90° \Leftrightarrow \vec{AB}\cdot\vec{AC} = 0$

$\Leftrightarrow -m^2 + 2m + 11 = 0$

$\Delta = 2^2 - 4\times(-1)\times 11 = 48 > 0$ (2 racines réelles)

$m = \frac{-2-\sqrt{48}}{-2} = \frac{-2-4\sqrt{3}}{-2} = 1 + 2\sqrt{3}$ ou $m = 1 - 2\sqrt{3}$

Ex4 : On se place dans le repère orthonormé $(A, \vec{i}, \vec{j})$.

donc $A(0;0)$, $B(5;0)$, $C(5;3)$, $D(0;3)$, $E(2{,}5;0)$

$\cos(\theta) = \frac{\vec{AC}\cdot\vec{DE}}{AC \times DE}$ avec $\vec{AC}\begin{pmatrix}5\\3\end{pmatrix}$ et $\vec{DE}\begin{pmatrix}2{,}5\\-3\end{pmatrix}$

donc $\vec{AC}\cdot\vec{DE} = 5\times 2{,}5 + 3\times(-3) = 3{,}5$

$AC = \sqrt{5^2+3^2} = \sqrt{34}$ et $DE = \sqrt{2{,}5^2+(-3)^2} = \sqrt{15{,}25}$

donc $\cos(\theta) = \frac{3{,}5}{\sqrt{34}\times\sqrt{15{,}25}} = \frac{3{,}5}{\sqrt{518{,}5}}$

donc $\theta = \cos^{-1}\left[\frac{3{,}5}{\sqrt{518{,}5}}\right] \simeq 81{,}16°$

COURS : 5 pts   Ex1 : 5 pts (*)   Ex2 : 4 pts (**)

Ex3 : 4 pts (**)   Ex4 : 2 pts (***)