Ex1 : 1) $\overrightarrow{AB}\cdot\overrightarrow{AC} = AB \times AC \times \cos(\hat{A}) = 3 \times 2 \times \cos(45°) = 6 \times \frac{\sqrt{2}}{2} = \boxed{3\sqrt{2}}$

2) $\overrightarrow{AB}\cdot\overrightarrow{AC} = \overrightarrow{AH}\cdot\overrightarrow{AB} = \boxed{5AH}$

3) $\overrightarrow{AB}\cdot\overrightarrow{AC} = \begin{pmatrix}1\\-2\end{pmatrix}\cdot\begin{pmatrix}-3\\-2\end{pmatrix} = -3+4 = \boxed{1}$

Ex2 : 1) $BD^2 = AB^2 + AD^2 = 3^2 + 4^2 = 25$ donc $BD = 5$

$\cos(\widehat{BEC}) = \frac{EC}{BE}$ et $\sin(\widehat{BEC}) = \frac{BC}{BE}$ donc $BE = \frac{4}{\sin(60°)}$

soit $BE = \frac{4}{\sqrt{3}/2} = \boxed{\frac{8}{\sqrt{3}}}$ (ou $\frac{8\sqrt{3}}{3}$) et $EC = \frac{8}{\sqrt{3}} \times \cos(60°) = \boxed{\frac{4}{\sqrt{3}}}$

2) $\overrightarrow{AB}\cdot\overrightarrow{AD} = \boxed{0}$, car $(AB)\perp(AD)$

$\overrightarrow{AD}\cdot\overrightarrow{CB} = AD \times CB \times \cos(180°) = 4 \times 4 \times (-1) = \boxed{-16}$

$\overrightarrow{BA}\cdot\overrightarrow{BE} = BA \times BE \times \cos(120°) = 3 \times \frac{8}{\sqrt{3}} \times (-0,5) = \frac{-12}{\sqrt{3}} = \boxed{-4\sqrt{3}}$

$\overrightarrow{DC}\cdot\overrightarrow{DB} = \overrightarrow{DC}\cdot\overrightarrow{DC} = DC \times DC \times \cos(0°) = 3 \times 3 \times 1 = \boxed{9}$

$\overrightarrow{EB}\cdot\overrightarrow{EC} = \overrightarrow{EC}\cdot\overrightarrow{EC} = EC \times EC \times \cos(0°) = \frac{4}{\sqrt{3}} \times \frac{4}{\sqrt{3}} = \boxed{\frac{16}{3}}$

$\overrightarrow{CB}\cdot\overrightarrow{BD} = CB \times BD \times \cos(\hat{B}) = CB \times BC \times \cos(180°) = 4 \times 4 \times (-1) = \boxed{-16}$

Ex3 : 1) $\vec{u} = k.\vec{v}$ ($k \in \mathbb{R}^*$) donc $\begin{pmatrix}3\\1-m\end{pmatrix} = k.\begin{pmatrix}2\\m+3\end{pmatrix}$

donc $3(m+3) = 2(1-m)$ donc $3m + 9 = 2 - 2m$ soit $\boxed{m = -1,4}$

2) $\vec{u}\perp\vec{v}$ donc $\vec{u}\cdot\vec{v} = 0$ donc $\begin{pmatrix}3\\1-m\end{pmatrix}\cdot\begin{pmatrix}2\\m+3\end{pmatrix} = 0$

donc $6 + (1-m)(m+3) = 0$

donc $-m^2 - 2m + 9 = 0$

soit $m^2 + 2m - 9 = 0$, on obtient $\Delta = 40 > 0$

Il existe donc 2 racines distinctes :

$m = \frac{-2-\sqrt{40}}{2} = \frac{-2-2\sqrt{10}}{2} = \boxed{-1-\sqrt{10}}$

ou $m = \frac{-2+\sqrt{40}}{2} = \boxed{-1+\sqrt{10}}$

Ex4 : 1) $\overrightarrow{DM}\cdot\overrightarrow{DN} = (\overrightarrow{DA} + \overrightarrow{AM})\cdot(\overrightarrow{DC} + \overrightarrow{CN})$

$= \overrightarrow{DA}\cdot\overrightarrow{DC} + \overrightarrow{AM}\cdot\overrightarrow{DC} + \overrightarrow{DA}\cdot\overrightarrow{CN} + \overrightarrow{AM}\cdot\overrightarrow{CN}$

$= 0 + \frac{1}{2}\overrightarrow{AB}\cdot\overrightarrow{DC} + \frac{1}{3}\overrightarrow{DA}\cdot\overrightarrow{CB} + 0$

$= \frac{1}{2} \times 6 \times 6 \times \cos(0°) + \frac{1}{3} \times 6 \times 6 \times \cos(0°) = \boxed{30}$

2) $DM^2 = DA^2 + AM^2 = 6^2 + 3^2 = 45$ donc $DM = \sqrt{45}$

soit $\boxed{DM = 3\sqrt{5}}$

$DN^2 = DC^2 + CN^2 = 6^2 + 2^2 = 40$

donc $DN = \sqrt{40}$ soit $\boxed{DN = 2\sqrt{10}}$

3) $\cos(\alpha) = \cos(\widehat{MDN}) = \frac{\overrightarrow{DM}\cdot\overrightarrow{DN}}{DM \times DN}$

donc $\cos(\alpha) = \frac{30}{3\sqrt{5} \times 2\sqrt{10}} = \frac{5}{\sqrt{50}} = \frac{5\sqrt{50}}{50} = \frac{\sqrt{50}}{10} = \frac{5\sqrt{2}}{10}$

donc $\cos(\alpha) = \frac{\sqrt{2}}{2}$ d'où $\boxed{\alpha = 45°}$