

# 7-6

## Volume of Pyramids and Cones

### MAIN IDEA

Find the volumes of pyramids and cones.

### New Vocabulary

**cone**

### Math Online

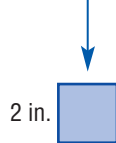
[glencoe.com](http://glencoe.com)

- Extra Examples
- Personal Tutor
- Self-Check Quiz

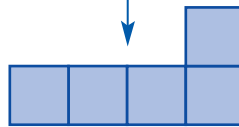
### MINI Lab

In this Mini Lab, you will investigate the relationship between the volume of a pyramid and the volume of a prism with the same base area and height.

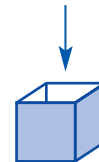
Draw and cut out 5 squares.



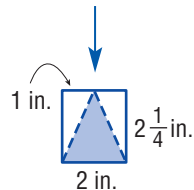
Tape together as shown.



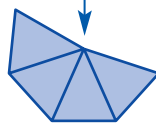
Fold and tape to form a cube with an open top.



Draw and cut out 4 isosceles triangles.



Tape together as shown.



Fold and tape to form an open square pyramid.



1. Compare the base areas and the heights of the two solids.
2. Fill the pyramid with rice, sliding a ruler across the top to level the amount. Pour the rice into the cube. Repeat until the prism is filled. How many times did you fill the pyramid in order to fill the cube?
3. What fraction of the cube's volume does one pyramid fill?

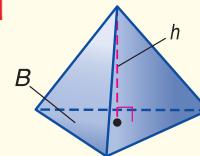
The volume of a pyramid is one-third the volume of a prism with the same base area and height.

### Volume of a Pyramid

#### Key Concept

**Words** The volume  $V$  of a pyramid is one-third the area of the base  $B$  times the height  $h$ .

**Model**



**Symbols**  $V = \frac{1}{3}Bh$

The height of a pyramid or cone is the distance from the vertex, perpendicular to the base.

## Study Tip

### Estimation

You can estimate the volume of the pyramid in Example 1 to be about

$$\frac{1}{3} \left( \frac{1}{2} \cdot 8 \cdot 6 \right) (11) \text{ or } 88 \text{ m}^3.$$

Since  $95.04 \text{ m}^3$  is close to  $88 \text{ m}^3$ , the answer is reasonable.

## EXAMPLE Find the Volume of a Pyramid

- 1 Find the volume of the pyramid. Round to the nearest tenth.

$$V = \frac{1}{3}Bh$$

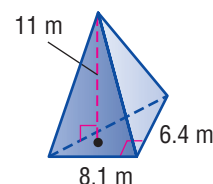
Volume of a pyramid

$$V = \frac{1}{3} \left( \frac{1}{2} \cdot 8.1 \cdot 6.4 \right) 11 \quad B = \frac{1}{2} \cdot 8.1 \cdot 6.4, h = 11$$

$$V = 95.04$$

Simplify.

The volume is about 95.0 cubic meters.



### CHECK Your Progress

- a. Find the volume of a pyramid that has a height of 5 yards and a square base with sides 2 yards long.

## Real-World EXAMPLE

- 2 **ARCHITECTURE** The Louvre Pyramid in Paris has a square base with sides 112 feet long. If the volume is 296,875 cubic feet, find the height of the pyramid.

$$V = \frac{1}{3}Bh$$

Volume of a pyramid

$$296,875 = \frac{1}{3} \cdot 12,544 \cdot h$$

Replace  $V$  with 296,875 and  $B$  with  $112 \cdot 112$  or 12,544.

$$296,875 = \frac{12,544}{3}h$$

Multiply.

$$\frac{3}{12,544} \cdot 296,875 = \frac{3}{12,544} \cdot \frac{12,544}{3}h$$

Multiply each side by  $\frac{3}{12,544}$ .

$$71 \approx h$$

Simplify.

The height of the pyramid is about 71 feet.

### CHECK Your Progress

- b. **CRAFTS** Nicco made a pyramid-shaped candle. The volume of the candle is 864 cubic centimeters and its base has an area of 144 square centimeters. How high is the candle?

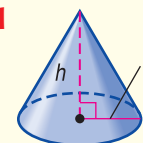
A **cone** is a three-dimensional figure with one circular base. A curved surface connects the base and the vertex. The volumes of a cone and a cylinder are related in the same way as those of a pyramid and prism.

## Volume of a Cone

### Key Concept

**Words** The volume  $V$  of a cone with radius  $r$  is one-third the area of the base  $B$  times the height  $h$ .

**Model**



**Symbols**  $V = \frac{1}{3}Bh$  or  $V = \frac{1}{3}\pi r^2h$

## EXAMPLE Find the Volume of a Cone

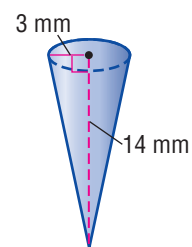
3 Find the volume of the cone.

$$V = \frac{1}{3}\pi r^2 h \quad \text{Volume of a cone}$$

$$V = \frac{1}{3} \cdot \pi \cdot 3^2 \cdot 14 \quad \text{Replace } r \text{ with } 3 \text{ and } h \text{ with } 14.$$

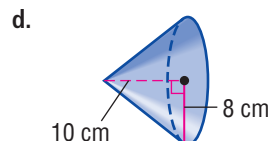
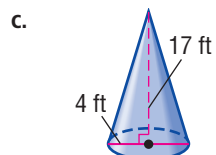
$$V \approx 131.9 \quad \text{Simplify. Use a calculator.}$$

The volume is about 131.9 cubic millimeters.



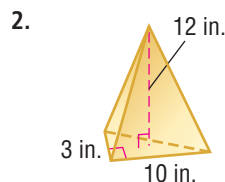
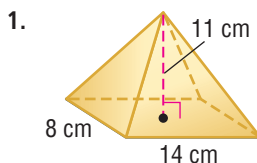
### CHECK Your Progress

Find the volume of each cone. Round to the nearest tenth.



## CHECK Your Understanding

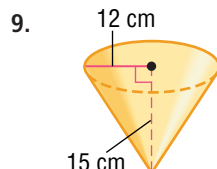
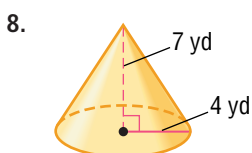
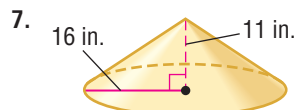
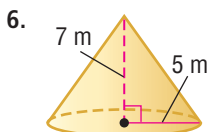
**Example 1** (p. 381) Find the volume of each pyramid. Round to the nearest tenth.



- Find the volume of a pyramid that has a height of 125 centimeters and a square base with sides 95 centimeters long.
- Find the volume of a pyramid that has a height of 17 feet and a square base with sides 22 feet long.

**Example 2** (p. 381) 5. **ARCHAEOLOGY** El Castillo, the pyramid at Chichen Itza in Mexico is 30 meters tall with a volume of about 30,580 cubic meters. What is the length of each side of the square base?

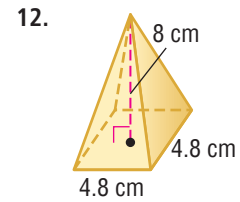
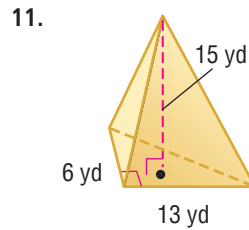
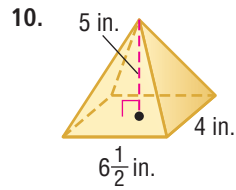
**Example 3** (p. 382) Find the volume of each cone. Round to the nearest tenth.



# Practice and Problem Solving

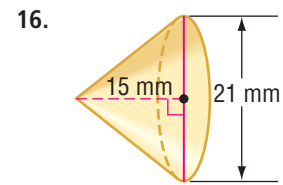
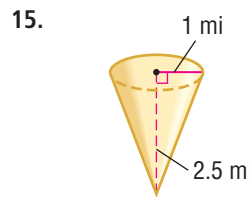
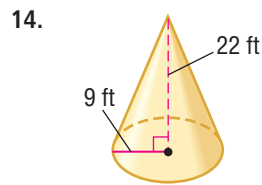
HOMEWORK HELP	
For Exercises	See Examples
10–13	1
18	2
14–17	3

Find the volume of each pyramid. Round to the nearest tenth.



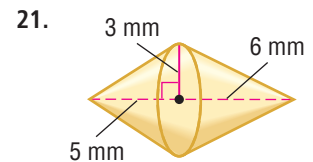
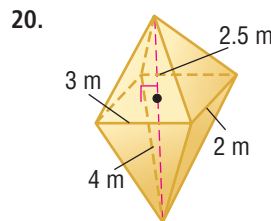
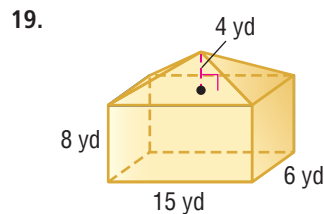
13. triangular pyramid: triangle base, 10 cm; triangle height, 7 cm; pyramid height, 15 cm

Find the volume of each cone. Round to the nearest tenth.

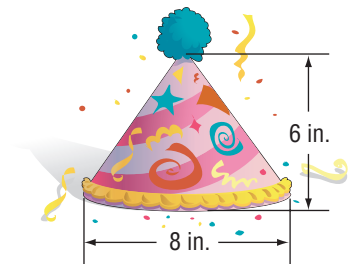


17. cone: diameter, 12 m; height, 5 m
18. **SCIENCE** A model of a volcano constructed for a science project is cone-shaped with a diameter of 8 inches. If the volume of the model is about 201 cubic inches, how tall is the model?

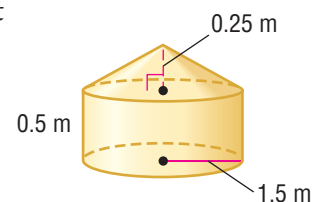
Find the volume of each solid. Round to the nearest tenth if necessary.



22. **HATS** A clown wants to fill his party hat with confetti. Use the drawing at the right to determine how much confetti his hat will hold.



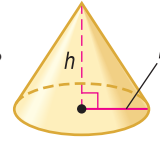
23. **IRRIGATION** A water tank like the one at the right is used to water flowers at a park. Water can be pumped from the tank at a rate of 25 liters per minute. How long will it take to use all of the water in a full tank? Round to the nearest minute. (*Hint*: 1 liter = 1,000 cm<sup>3</sup>)



**EXTRA PRACTICE**  
See pages 687, 706.

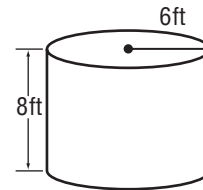
**H.O.T. Problems**

24. **CHALLENGE** How could you change the height of a cone so that its volume would remain the same when its radius was tripled?
25. **OPEN ENDED** Draw and label a triangular pyramid with a volume of 36 cubic centimeters.
26. **NUMBER SENSE** Which would have a greater effect on the volume of a cone, doubling its radius or doubling its height? Explain your reasoning.
27. **WRITING IN MATH** Write about a real-world situation that can be solved by finding the volume of a cone.

**TEST PRACTICE**

28. A rectangular pyramid has a base 18 inches by 30 inches and a height of 36 inches. Which is closest to the volume of the pyramid in cubic feet?
- A  $2.5 \text{ ft}^3$   
 B  $3 \text{ ft}^3$   
 C  $4 \text{ ft}^3$   
 D  $5.5 \text{ ft}^3$

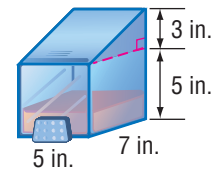
29. Find the volume of the cylinder. Round to the nearest tenth if necessary.



- F  $48 \text{ ft}^3$       H  $288 \text{ ft}^3$   
 G  $150.3 \text{ ft}^3$       J  $904.8 \text{ ft}^3$

**Spiral Review**

30. **DISPENSER** Find the volume of the soap dispenser at the right. (Lesson 7-5)
31. Name the number and shapes of the faces of a trapezoidal prism. Then name the number of edges and vertices. (Lesson 7-4)
32. **GEOMETRY** Graph triangle  $ABC$  with vertices  $A(1, 2)$ ,  $B(4, -1)$ , and  $C(2, -4)$ . Then graph its image after a reflection over the  $y$ -axis, and write the coordinates of the image's vertices. (Lesson 6-6)
33. **SHOPPING** Etu saved \$90 when he purchased a DVD recorder on sale. If the sale price was 37.5% off the regular price, what was the regular price of the DVD recorder? (Lesson 5-7)

**GET READY for the Next Lesson**

**PREREQUISITE SKILL** Find the circumference of each circle. Round to the nearest tenth. (Lesson 7-1)

34. diameter, 9 in.      35. diameter, 5.5 ft      36. radius, 2 m      37. radius, 3.8 cm