Thales' theorem

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Let's draw a circle from the central point O, and draw a diameter AB.
Chose a point C lying on the circle, and connect it with A and B. The circle is circumscripted to the ABC triangle, and point O is the medium point of AB side.Connecting O to C, we observe that OA = OC = radius of the circle, so triangle AOC is isosceles, and so angle CAO^ = ACO^ = α .
Doing the same observations with BOC triangle, OC = OB = radius, so OBC is an isosceles triangle too, and so OCB^ = CBO^ = β .

Knowing that the sum of the internal angles of a triangle is 180°:
CAB^ + ABC^ + BCA^ = 180° = $\alpha + \beta + (\alpha + \beta) = 180°$ $2 (\alpha + \beta) = 180°$ $\alpha + \beta = 90°$
So triangle ABC is rectangle!!!!! We've arrived at Thales' theorem:
If we connect the extremes of a diameter of a circle to one point lying on the circle, we always obtain a rectangle triangle, which hypotenuse is the diameter AB.

Thales' theorem II (reverse) : if the central point of a circle circumscripted to a triangle is the medium point of one of its sides, the triangle is rectangle, and the central point of the circle is the medium point of the hypotenuse.

Example 1 : we've a circle, with central point O, and an external point P. We want to draw the tangents to the circle from point P.I connect O to P. I draw the medium point of OP, F (with compasses!!!). I draw a circle from central point F, with radius FO.This new circle meets the first one at points E and G. Connecting P to G and E I obtain the tangent lines.
I observe that triangles POE and POG are unscripted into the second circle, and their hypotenuse is the diameter OP. For Thales' theorem these triangles are rectangle!
If I draw the two tangent lines to a circle from an external point, the lines are equal and the radius of the circle is perpendicular to them.

Example 2 : Prove that the vertices of a side of a triangle and the intersection point with the altitude drawn by these point with the opposite side, are lying on the same circle.
Let's draw triangle ABC, and then the altitudes from A and B, AQ and BT. We know that BT and AQ are perpendicular to AC and CB. So ABT and ABQ are rectangle triangle, and they've the same hypotenuse AB. From Thales' theorem, AB is the diameter of the circle, which passes by T and Q.

Exercise 1 : Imagine a circle road, we have a town A far 13 km from town B, both are on the road, and it's the maximum distance two town can have on this road. If a bike goes straight from A to town C, on the same road, and this distance is 5 km, how far is C from B?

Exercise 2 : We have a circle of 5 cm radius. If we inscribe a triangle, and one rectangle angle side is: a) 6 cm ; b) 8 cm ; c) 2 cm ; how much is the other rectangle angle side long?

Exercise 3 : We've a circle with central point O and radius 4 cm. Taking a point P of the plane far from O : a) 5 cm ; b) 7 cm ; c) 12 cm ; let's calculate the length of the lines tangent to the circle from point P.