Dossier d'exercices corrigés - Chapitre 7

1 **a)** $f(x) = 2x + 1$. **b)** $f(x) = 3x^2 + x - 4$.

a) $F(x) = x^2 + x + c \quad c \in \mathbb{R}$

b) $F(x) = 3\dfrac{x^3}{3} + \dfrac{x^2}{2} - 4x + c$

$F(x) = x^3 + \dfrac{x^2}{2} - 4x + c$

2 **a)** $f(x) = 4x^3 - x^2 + \dfrac{5}{2}$.

$F(x) = 4\dfrac{x^4}{4} - \dfrac{x^3}{3} + \dfrac{5}{2}x + c$

$F(x) = x^4 - \dfrac{x^3}{3} + \dfrac{5}{2}x + c$

b) $f(x) = \dfrac{3}{4}x^2 - \dfrac{2}{3}x + 4$.

$F(x) = \dfrac{3}{4}\dfrac{x^3}{3} - \dfrac{2}{3}\dfrac{x^2}{2} + 4x + c$

$F(x) = \dfrac{x^3}{4} - \dfrac{x^2}{3} + 4x + c$

3 **a)** $f(x) = \dfrac{x^2}{2} - \dfrac{x}{3}$.

$F(x) = \dfrac{1}{2} \times \dfrac{x^3}{3} - \dfrac{1}{3} \times \dfrac{x^2}{2} + c$

$F(x) = \dfrac{x^3}{6} - \dfrac{x^2}{6} + c$

b) $f(x) = \dfrac{2x^3}{3} - \dfrac{5x}{4} + 1$.

$F(x) = \dfrac{2}{3} \times \dfrac{x^4}{4} - \dfrac{5}{4} \times \dfrac{x^2}{2} + x + c$

$F(x) = \dfrac{x^4}{6} - \dfrac{5}{8}x^2 + x + c$

4 **a)** $f(x) = 1 - \dfrac{2}{5}x^2 + x^3$.

$F(x) = x - \dfrac{2}{5} \times \dfrac{x^3}{3} + \dfrac{x^4}{4} + c$

$F(x) = x - \dfrac{2x^3}{15} + \dfrac{x^4}{4} + c$

b) $f(x) = \sqrt{2} - \dfrac{x^5}{5}$.

$F(x) = x\sqrt{2} - \dfrac{1}{5}\dfrac{x^6}{6} + c$

$F(x) = x\sqrt{2} - \dfrac{x^6}{30} + c$

5 **a)** $f(x) = 0{,}8x^3 - 1{,}2x^2 + 0{,}5$.

$F(x) = 0{,}8\dfrac{x^4}{4} - 1{,}2\dfrac{x^3}{3} + 0{,}5x + c$

$F(x) = 0{,}2x^4 - 0{,}4x^3 + 0{,}5x + c$

b) $f(x) = 0{,}2x^4 - 3{,}6x^3 - 8{,}2x$.

$F(x) = 0{,}2\dfrac{x^5}{5} - 3{,}6\dfrac{x^4}{4} - 8{,}2\dfrac{x^2}{2} + c$

$F(x) = 0{,}04x^5 - 0{,}9x^4 - 4{,}1x^2 + c$

Pour les exercices 6 à 11

Trouvez une primitive sur $]0\,;+\infty[$ de la fonction f.

6 **a)** $f(x) = \dfrac{1}{x^3}$.

$F(x) = \dfrac{-1}{2x^2} + C$

b) $f(x) = \dfrac{3}{x^4}$.

$F(x) = 3x\dfrac{-1}{3x^3} + C = \dfrac{-1}{x^3} + C$

7 a) $f(x) = \frac{2}{x^3} + \frac{5}{x^2}$.

$F(x) = 2 \times \frac{-1}{2x^2} + 5 \times \frac{-1}{x} + c = \frac{-1}{x^2} - \frac{5}{x} + c$

b) $f(x) = \frac{3}{5x^4} - \frac{7}{x^3}$.

$F(x) = \frac{3}{5} \times \frac{-1}{3x^3} - 7 \times \frac{-1}{2x^2} + c = \frac{-1}{5x^3} + \frac{7}{2x^2} + c$

8 a) $f(x) = \frac{3}{2\sqrt{x}} + \frac{1}{x^2}$. $= 3 \times \left(\frac{1}{2\sqrt{x}}\right) - \left(\frac{-1}{x^2}\right)$

$F(x) = 3\sqrt{x} - \frac{1}{x} + c$

b) $f(x) = \frac{6}{x^4} - \frac{1}{\sqrt{x}}$. $= 6 \times \frac{1}{x^4} - 2 \times \left(\frac{1}{2\sqrt{x}}\right)$

$F(x) = 6 \times -\frac{1}{3x^3} - 2\sqrt{x} + c$

9 a) $f(x) = \frac{1}{x} - \frac{3}{x^2}$.

$F(x) = \ln x + \frac{3}{x} + c$

b) $f(x) = 4e^x - \frac{1}{x}$.

$F(x) = 4e^x - \ln x + c$

10 a) $f(x) = \frac{e^x}{3} - \frac{1}{2x}$. $= \frac{e^x}{3} - \frac{1}{2} \times \frac{1}{x}$

d'où $F(x) = \frac{e^x}{3} - \frac{1}{2} \ln x + c$

b) $f(x) = -\frac{2e^x}{3} + \frac{3}{4x^2}$. $= -\frac{2e^x}{3} - \frac{3}{4}\left(-\frac{1}{x^2}\right)$

$F(x) = -\frac{2e^x}{3} - \frac{3}{4x} + c$

11 a) $f(x) = -\frac{1}{2x^3} + \pi x$. $= -\frac{1}{2} \times \left(\frac{1}{x^3}\right) + \pi x$

$F(x) = -\frac{1}{2} \times \frac{-1}{2x^2} + \frac{\pi x^2}{2} + c$

$F(x) = \frac{1}{4x^2} + \frac{\pi x^2}{2} + c$

11 b) $f(x) = \frac{5}{3x^4} - \frac{1}{\sqrt{2}x}$. $= \frac{5}{3} \times \frac{1}{x^4} - \frac{1}{\sqrt{2}} \times \frac{1}{x}$

$F(x) = \frac{5}{3} \times \frac{-1}{3x^3} - \frac{1}{\sqrt{2}} \ln x + c = \frac{-5}{9x^3} - \frac{1}{\sqrt{2}} \ln x + c$

Pour les exercices 12 à 13

Trouvez la primitive de la fonction f qui s'annule pour $x = 0$.

12 **a)** $f(x) = x^3 + x + 1$.

$F(x) = \frac{x^4}{4} + \frac{x^2}{2} + x + C$

$F(0) = 0 + 0 + 0 + C$

$F(0) = C$

ainsi $F(0) = 0 \iff C = 0$

$$\boxed{F(x) = \frac{x^4}{4} + \frac{x^2}{2} + x}$$

b) $f(x) = 3x^3 - 2x + 5$.

$F(x) = \frac{3}{4}x^4 - x^2 + 5x + C$

$F(0) = 0 - 0 + 0 + C = C$

ainsi $F(0) = 0 \iff C = 0$

$$\boxed{F(x) = \frac{3x^4}{4} - x^2 + 5x}$$

13 **a)** $f(x) = e^x - x$.

$F(x) = e^x - \frac{x^2}{2} + C$

$F(0) = e^0 - 0 + C = 1 + C$

$F(0) = 0 \iff 1 + C = 0$

$\iff C = -1$

$$\boxed{F(x) = e^x - \frac{x^2}{2} - 1}$$

13 **b)** $f(x) = 2e^x - \frac{1}{2}$.

$F(x) = 2e^x - \frac{1}{2}x + c$

$F(0) = 2e^0 - 0 + c = 2 + c$

$\boxed{F(x) = 2e^x - \frac{1}{2}x - 2}$

$F(0) = 0 \Leftrightarrow 2 + c = 0$

$\Leftrightarrow \quad c = -2$

14 Trouvez la primitive, sur $]0\,;\,+\infty[$, de la fonction $f : x \mapsto 6x^2 - \frac{3}{x} - 1$ qui s'annule pour $x = 1$.

$F(x) = \frac{6x^3}{3} - 3\ln x - x + c$

$F(x) = 2x^3 - 3\ln x - x + c$

$F(1) = 2 - 3 \times 0 - 1 + c = 1 + c$

$F(1) = 0 \Leftrightarrow 1 + c = 0 \quad \Leftrightarrow \quad c = -1$

$\boxed{F(x) = 2x^3 - 3\ln x - x - 1}$

15 Trouvez la primitive, sur $]0\,;\,+\infty[$, de la fonction $f : x \mapsto \frac{3}{x^2} - \frac{5}{x}$ qui prend la valeur $2 - \frac{1}{e}$ en $x = e$.

$F(x) = 3x^{-1} - 5\ln x + c = -\frac{3}{x} - 5\ln x + c$

$F(e) = -\frac{3}{e} - 5 \times 1 + c = -\frac{3}{e} - 5 + c$

$F(e) = 2 - \frac{1}{e}$

$\Leftrightarrow -\frac{3}{e} - 5 + c = 2 - \frac{1}{e}$

$\Leftrightarrow c = 7 + \frac{2}{e}$

$\boxed{F(x) = -\frac{3}{x} - 5\ln x + 7 + \frac{2}{e}}$