

on souhaite inverser la Matrice :

$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}.$$

$$\left( \begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 \end{array} \right)$$

$$\begin{array}{l} L_2 - L_1 \rightarrow L_2 \\ L_3 - 2L_1 \rightarrow L_3 \end{array} \left( \begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & -1 & -3 & -2 & 0 & 1 \end{array} \right)$$

$$L_3 + L_2 \rightarrow L_3 \left( \begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & -4 & -3 & 1 & 1 \end{array} \right)$$

$$\frac{-1}{4}L_3 \rightarrow L_3 \left( \begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 3/4 & -1/4 & -1/4 \end{array} \right)$$

$$\begin{array}{l} L_1 - 2L_3 \rightarrow L_1 \\ L_2 + L_3 \rightarrow L_2 \end{array} \left( \begin{array}{ccc|ccc} 1 & 1 & 0 & -1/2 & 1/2 & 1/2 \\ 0 & 1 & 0 & -1/4 & 3/4 & -1/4 \\ 0 & 0 & 1 & 3/4 & -1/4 & -1/4 \end{array} \right)$$

$$L_1 - L_2 \rightarrow L_1 \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & -1/4 & -1/4 & 3/4 \\ 0 & 1 & 0 & -1/4 & 3/4 & -1/4 \\ 0 & 0 & 1 & 3/4 & -1/4 & -1/4 \end{array} \right)$$

La matrice inverse est donc

$$\begin{pmatrix} -1/4 & -1/4 & 3/4 \\ -1/4 & 3/4 & -1/4 \\ 3/4 & -1/4 & -1/4 \end{pmatrix}$$

on souhaite inverser la Matrice :

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 2 \\ 0 & 1 & 1 \end{pmatrix} \begin{array}{l} L_1 \\ L_2 \\ L_3 \end{array} \quad I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 2 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 0 & 1 & 1 \end{pmatrix} \begin{array}{l} L_1 \\ L_2 \leftarrow L_2 - 2L_1 \\ L_3 \end{array} \quad \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 3 \end{pmatrix} \begin{array}{l} L_1 \\ L_2 \\ L_3 \leftarrow L_3 + L_2 \end{array} \quad \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -2 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{array}{l} L_1 \\ L_2 \leftarrow L_2 - \frac{2}{3}L_3 \\ L_3 \end{array} \quad \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & -\frac{2}{3} \\ -2 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{array}{l} L_1 \leftarrow L_1 + L_2 \\ L_2 \\ L_3 \end{array} \quad \begin{pmatrix} 1 & 1 & -\frac{2}{3} \\ \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ -2 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{array}{l} L_2 \leftarrow -L_2 \\ L_3 \leftarrow \frac{1}{3}L_3 \end{array} \quad \begin{pmatrix} 1 & 1 & -\frac{2}{3} \\ \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$