8.6 Proportions and Similar Triangles

What you should learn

GOAL 1 Use proportionality theorems to calculate segment lengths.

GOAL 2 To solve **real-life** problems, such as determining the dimensions of a piece of land in **Exs. 29 and 30**.

Why you should learn it

▼ Model **real-life** situations using proportionality theorems, as in the construction problem in **Example 5**.

GOAL 1 USING PROPORTIONALITY THEOREMS

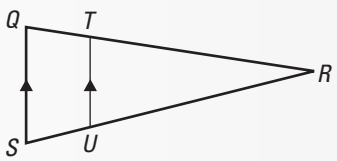
In this lesson, you will study four proportionality theorems. Similar triangles are used to prove each theorem. You are asked to prove the theorems in Exercises 31–33 and 38.

THEOREMS

THEOREM 8.4 *Triangle Proportionality Theorem*

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

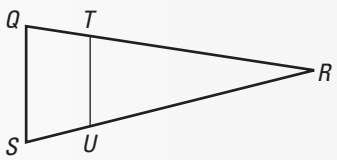
If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.



THEOREM 8.5 *Converse of the Triangle Proportionality Theorem*

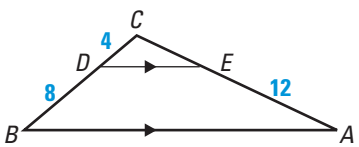
If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.



EXAMPLE 1 *Finding the Length of a Segment*

In the diagram $\overline{AB} \parallel \overline{ED}$, $BD = 8$, $DC = 4$, and $AE = 12$. What is the length of $\overline{EC}$?



SOLUTION

$\frac{DC}{BD} = \frac{EC}{AE}$ Triangle Proportionality Theorem

$\frac{4}{8} = \frac{EC}{12}$ Substitute.

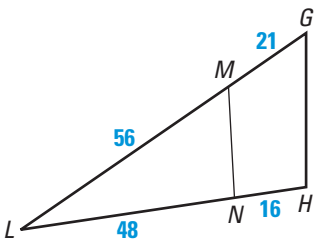
$\frac{4(12)}{8} = EC$ Multiply each side by 12.

$6 = EC$ Simplify.

▶ So, the length of $\overline{EC}$ is 6.

EXAMPLE 2 Determining Parallels

Given the diagram, determine whether $\overline{MN} \parallel \overline{GH}$.



SOLUTION

Begin by finding and simplifying the ratios of the two sides divided by $\overline{MN}$.

$$\frac{LM}{MG} = \frac{56}{21} = \frac{8}{3} \qquad \frac{LN}{NH} = \frac{48}{16} = \frac{3}{1}$$

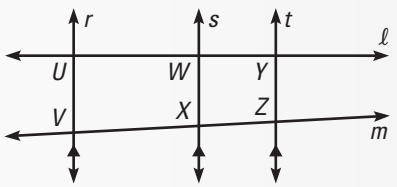
Because $\frac{8}{3} \neq \frac{3}{1}$, $\overline{MN}$ is not parallel to $\overline{GH}$.

THEOREMS

THEOREM 8.6

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

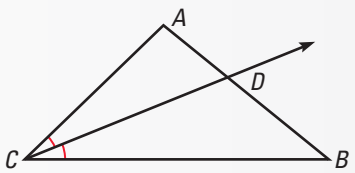
If $r \parallel s$ and $s \parallel t$, and ℓ and m intersect r, s, and t, then $\frac{UW}{WY} = \frac{VX}{XZ}$.



THEOREM 8.7

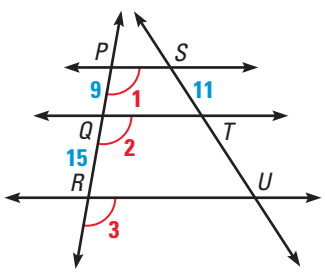
If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.

If $\overrightarrow{CD}$ bisects $\angle ACB$, then $\frac{AD}{DB} = \frac{CA}{CB}$.



EXAMPLE 3 Using Proportionality Theorems

In the diagram, $\angle 1 \cong \angle 2 \cong \angle 3$, and $PQ = 9$, $QR = 15$, and $ST = 11$. What is the length of $\overline{TU}$?



SOLUTION

Because corresponding angles are congruent the lines are parallel and you can use Theorem 8.6.

$\frac{PQ}{QR} = \frac{ST}{TU}$ — Parallel lines divide transversals proportionally.

$\frac{9}{15} = \frac{11}{TU}$ — Substitute.

$9 \cdot TU = 15 \cdot 11$ — Cross product property

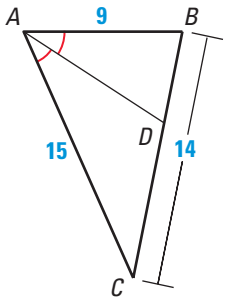
$TU = \frac{15(11)}{9} = \frac{55}{3}$ — Divide each side by 9 and simplify.

▶ So, the length of $\overline{TU}$ is $\frac{55}{3}$, or $18\frac{1}{3}$.

Using Algebra

EXAMPLE 4 Using Proportionality Theorems

In the diagram, $\angle CAD \cong \angle DAB$. Use the given side lengths to find the length of $\overline{DC}$.



SOLUTION

Since $\overline{AD}$ is an angle bisector of $\angle CAB$, you can apply Theorem 8.7.

Let $x = DC$. Then, $BD = 14 - x$.

$$\frac{AB}{AC} = \frac{BD}{DC}$$ Apply Theorem 8.7.

$$\frac{9}{15} = \frac{14 - x}{x}$$ Substitute.

$$9 \cdot x = 15(14 - x)$$ Cross product property

$$9x = 210 - 15x$$ Distributive property

$$24x = 210$$ Add $15x$ to each side.

$$x = 8.75$$ Divide each side by 24.

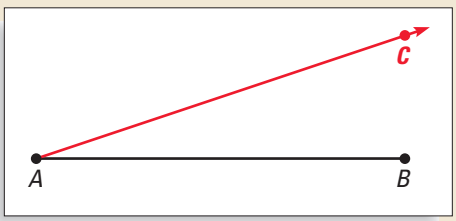
▶ So, the length of $\overline{DC}$ is 8.75 units.

ACTIVITY

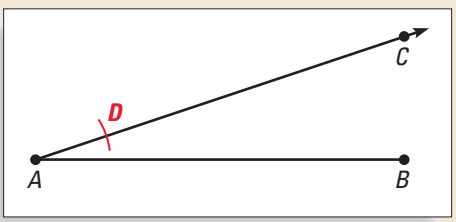
Construction

Dividing a Segment into Equal Parts (4 shown)

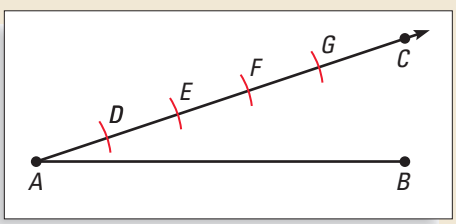
1. Draw a line segment that is about 3 inches long. Label the endpoints A and B. Choose any point C not on $\overleftrightarrow{AB}$. Draw $\overrightarrow{AC}$.



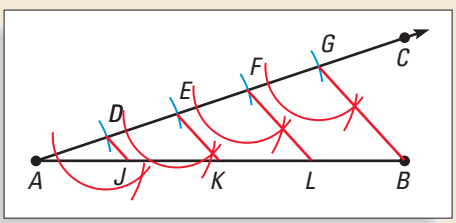
2. Using any length, place the compass point at A and make an arc intersecting $\overrightarrow{AC}$ at D.



3. Using the same compass setting, make additional arcs on $\overrightarrow{AC}$. Label the points E, F, and G so that $AD = DE = EF = FG$.



4. Draw $\overline{GB}$. Construct a line parallel to $\overline{GB}$ through D. Continue constructing parallel lines and label the points as shown. Explain why $AJ = JK = KL = LB$.



GOAL 2 USING PROPORTIONALITY THEOREMS IN REAL LIFE

EXAMPLE 5 *Finding the Length of a Segment*

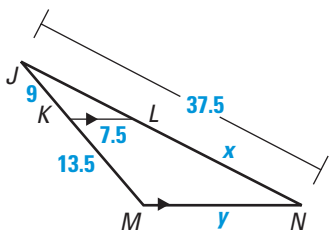
BUILDING CONSTRUCTION You are insulating your attic, as shown. The vertical 2×4 studs are evenly spaced. Explain why the diagonal cuts at the tops of the strips of insulation should have the same lengths.

SOLUTION

Because the studs $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ are each vertical, you know that they are parallel to each other. Using Theorem 8.6, you can conclude that $\frac{DE}{EF} = \frac{AB}{BC}$. Because the studs are evenly spaced, you know that $DE = EF$. So, you can conclude that $AB = BC$, which means that the diagonal cuts at the tops of the strips have the same lengths.

EXAMPLE 6 *Finding Segment Lengths*

In the diagram $\overline{KL} \parallel \overline{MN}$. Find the values of the variables.



SOLUTION

To find the value of x, you can set up a proportion.

$$\frac{9}{13.5} = \frac{37.5 - x}{x}$$ Write proportion.

$$13.5(37.5 - x) = 9x$$ Cross product property

$$506.25 - 13.5x = 9x$$ Distributive property

$$506.25 = 22.5x$$ Add $13.5x$ to each side.

$$22.5 = x$$ Divide each side by 22.5.

Since $\overline{KL} \parallel \overline{MN}$, $\triangle JKL \sim \triangle JMN$ and $\frac{JK}{JM} = \frac{KL}{MN}$.

$$\frac{9}{13.5 + 9} = \frac{7.5}{y}$$ Write proportion.

$$9y = 7.5(22.5)$$ Cross product property

$$y = 18.75$$ Divide each side by 9.

GUIDED PRACTICE

Vocabulary Check ✓

1. Complete the following: If a line divides two sides of a triangle proportionally, then it is __?__ to the third side. This theorem is known as the __?__.

Concept Check ✓

2. In $\triangle ABC$, $\overrightarrow{AR}$ bisects $\angle CAB$. Write the proportionality statement for the triangle that is based on Theorem 8.7.

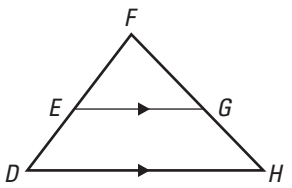
Determine whether the statement is *true* or *false*. Explain your reasoning.

3. $\frac{FE}{ED} = \frac{FG}{GH}$

4. $\frac{FE}{FD} = \frac{FG}{FH}$

5. $\frac{EG}{DH} = \frac{EF}{DF}$

6. $\frac{ED}{FE} = \frac{EG}{DH}$



Skill Check ✓

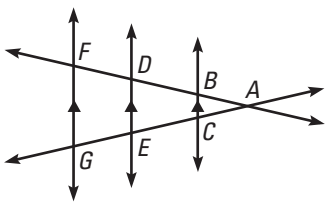
Use the figure to complete the proportion.

7. $\frac{BD}{BF} = \frac{?}{CG}$

8. $\frac{AE}{CE} = \frac{?}{BD}$

9. $\frac{?}{GA} = \frac{FD}{FA}$

10. $\frac{GA}{?} = \frac{FA}{DA}$



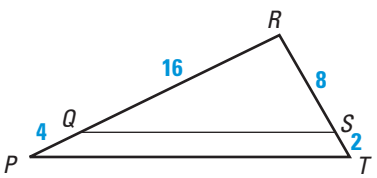
PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 818.

LOGICAL REASONING **Determine whether the given information implies that $\overline{QS} \parallel \overline{PT}$. Explain.**

11.



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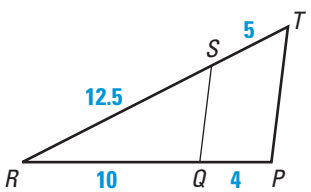

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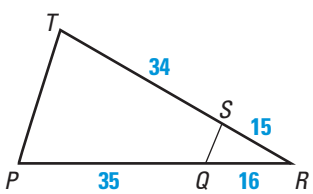

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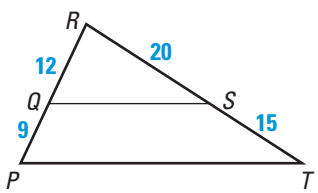


STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 21–28
Example 2: Exs. 11–20
Example 3: Exs. 21–28
Example 4: Exs. 21–28
Example 5: Exs. 29, 30, 36, 37
Example 6: Exs. 34–37

LOGICAL REASONING **Use the diagram shown to decide if you are given enough information to conclude that $\overline{LP} \parallel \overline{MQ}$. If so, state the reason.**

15. $\frac{NM}{ML} = \frac{NQ}{QP}$

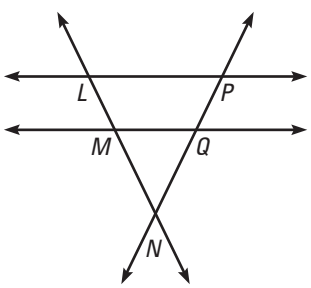
16. $\angle MNQ \cong \angle LNP$

17. $\angle NLP \cong \angle NMQ$

18. $\angle MQN \cong \angle LPN$

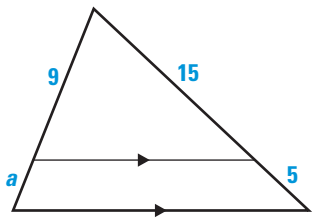
19. $\frac{LM}{MN} = \frac{LP}{MQ}$

20. $\triangle LPN \sim \triangle MQN$



USING PROPORTIONALITY THEOREMS Find the value of the variable.

21.



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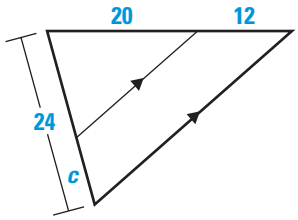

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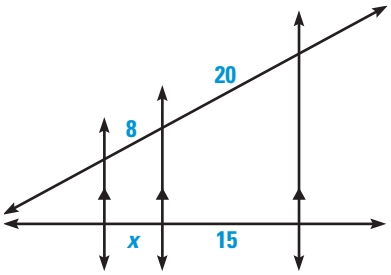

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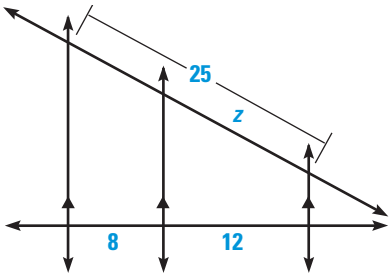
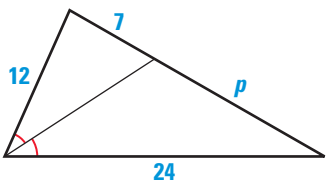


USING ALGEBRA Find the value of the variable.

25.



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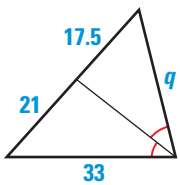

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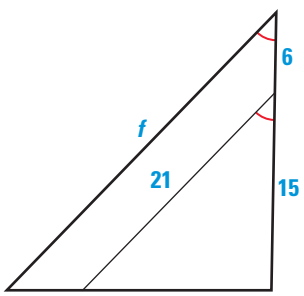

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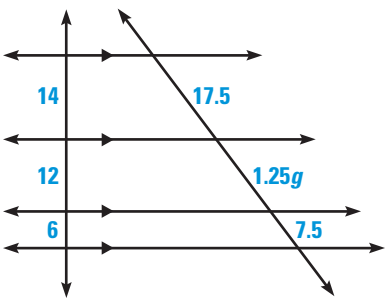
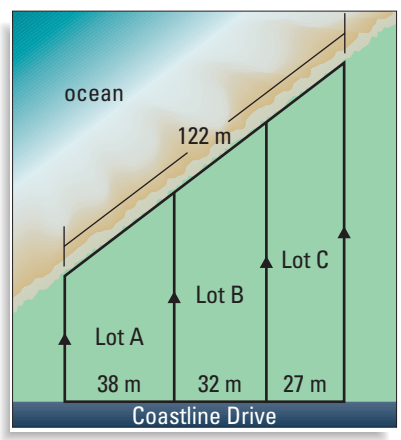


LOT PRICES **The real estate term for the distance along the edge of a piece of property that touches the ocean is "ocean frontage."**

29. Find the ocean frontage (to the nearest tenth of a meter) for each lot shown.

30. CRITICAL THINKING In general, the more ocean frontage a lot has, the higher its selling price. Which of the lots should be listed for the highest price?



FOCUS ON CAREERS

REAL ESTATE SALESPERSON

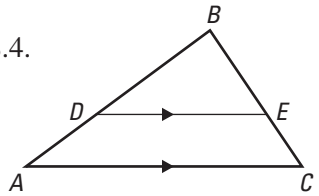
A real estate salesperson can help a seller establish a price for their property as discussed in Exercise 30.

CAREER LINK
www.mcdougallittell.com

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with proofs in Exs. 31–33.

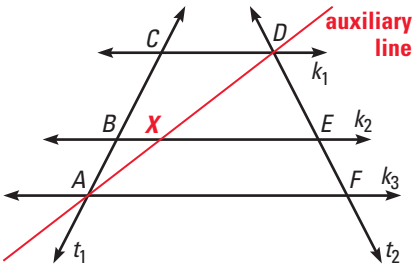
31. TWO-COLUMN PROOF Use the diagram shown to write a two-column proof of Theorem 8.4.



GIVEN ▶ $\overline{DE} \parallel \overline{AC}$

PROVE ▶ $\frac{DA}{BD} = \frac{EC}{BE}$

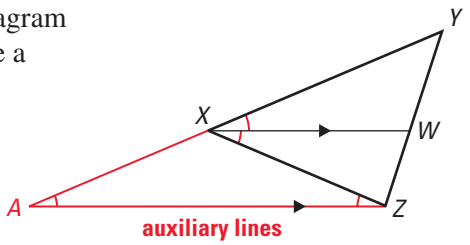
32. PARAGRAPH PROOF Use the diagram with the auxiliary line drawn to write a paragraph proof of Theorem 8.6.



GIVEN ▶ $k_1 \parallel k_2, k_2 \parallel k_3$

PROVE ▶ $\frac{CB}{BA} = \frac{DE}{EF}$

33. PARAGRAPH PROOF Use the diagram with the auxiliary lines drawn to write a paragraph proof of Theorem 8.7.

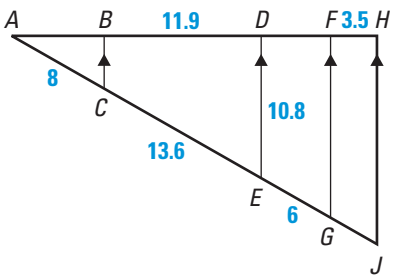


GIVEN ▶ $\angle YXW \cong \angle WXZ$

PROVE ▶ $\frac{YW}{WZ} = \frac{XY}{XZ}$

FINDING SEGMENT LENGTHS **Use the diagram to determine the lengths of the missing segments.**

34.



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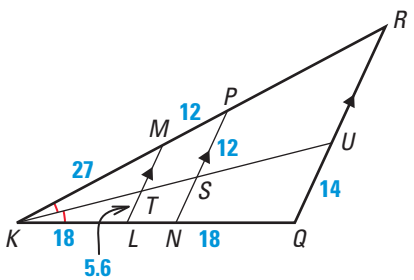


NEW YORK CITY **Use the following information and the map of New York City.** On Fifth Avenue, the distance between E 33rd Street and E 24th Street is about 2600 feet. The distance between those same streets on Broadway is about 2800 feet. All numbered streets are parallel.

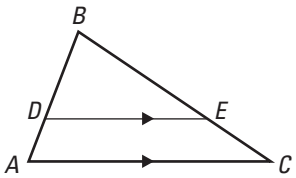
36. On Fifth Avenue, the distance between E 24th Street and E 29th Street is about 1300 feet. What is the distance between these two streets on Broadway?

37. On Broadway, the distance between E 33rd Street and E 30th Street is about 1120 feet. What is the distance between these two streets on Fifth Avenue?

38. *Writing* Use the diagram given for the proof of Theorem 8.4 from Exercise 31 to explain how you can prove the Triangle Proportionality Converse, Theorem 8.5.

Test Preparation

39. **MULTI-STEP PROBLEM** Use the diagram shown.



a. If $DB = 6$, $AD = 2$, and $CB = 20$, find EB.

b. Use the diagram to state three correct proportions.

c. If $DB = 4$, $AB = 10$, and $CB = 20$, find CE.

d. *Writing* Explain how you know that $\triangle ABC$ is similar to $\triangle DBE$.

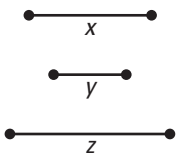
Challenge

40. **CONSTRUCTION** Perform the following construction.

GIVEN ▶ Segments with lengths x, y, and z

CONSTRUCT ▶ A segment of length p, such that $\frac{x}{y} = \frac{z}{p}$.

(*Hint:* This construction is like the construction on page 500.)



MIXED REVIEW

USING THE DISTANCE FORMULA **Find the distance between the two points.** (Review 1.3)

41. $A(10, 5)$, $B(-6, -4)$

42. $A(7, -3)$, $B(-9, 4)$

43. $A(-1, -9)$, $B(6, -2)$

44. $A(0, 11)$, $B(-5, 2)$

45. $A(0, -10)$, $B(4, 7)$

46. $A(8, -5)$, $B(0, 4)$

USING THE DISTANCE FORMULA **Place the figure in a coordinate plane and find the requested information.** (Review 4.7)

47. Draw a right triangle with legs of 12 units and 9 units. Find the length of the hypotenuse.

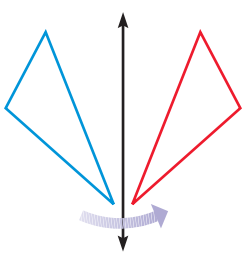
48. Draw a rectangle with length 16 units and width 12 units. Find the length of a diagonal.

49. Draw an isosceles right triangle with legs of 6 units. Find the length of the hypotenuse.

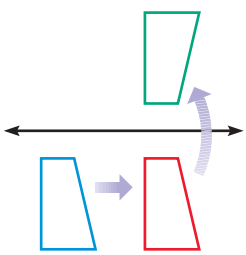
50. Draw an isosceles triangle with base of 16 units and height of 6 units. Find the length of the legs.

TRANSFORMATIONS **Name the type of transformation.** (Review 7.1– 7.3, 7.5 for 8.7)

51.



52.



53.