Ex1 a) $\vec{AB} \cdot \vec{AC} = -16$ b) $\vec{AB} \cdot \vec{AC} = -AB \times AC = -16$

$(\vec{AB}+\vec{AC})^2 = AB^2 + AC^2 + 2\vec{AB}\cdot\vec{AC}$

$\Leftrightarrow \vec{AB}\cdot\vec{AC} = \frac{1}{2}\left[(\vec{AB}+\vec{AC})^2 - AB^2 - AC^2\right] = \frac{1}{2}\left[(\vec{AD})^2 - AB^2 - AC^2\right] = -16$

Ex2 a) $\vec{AB}\cdot\vec{AC} = -AB \times AC \times \cos(\widehat{BAC}) = -12 \times 0{,}5 = -6$

b) $\vec{AB}\cdot\vec{AC} = AB \times AC \times \cos(30) = 9 \times \frac{\sqrt{3}}{2} = 4{,}5\sqrt{3}$

c) $\vec{AB}\cdot\vec{AC} = AB^2 = 4$

Ex3 a) $\vec{AB}\cdot\vec{AC} = AB^2 = 9$ b) $\vec{AB}\begin{pmatrix}-3\\-2\end{pmatrix}\cdot\vec{AC}\begin{pmatrix}1\\-2\end{pmatrix} = -3+4 = 1$

Ex4 1) $\vec{OI}\cdot\vec{OJ} = OI \times OJ = 6$ $\vec{OI}\cdot\vec{OK} = -OI \times OK = -6$ $\vec{OI}\cdot\vec{OB} = -OI \times OB \times \cos 120 = -3$

$\vec{OB}\cdot\vec{OA} = -OB \times OA \cos(150)$ ou $-OA \times OB \cos 30 = -6 \times \frac{\sqrt{3}}{2} = -3\sqrt{3}$

2) Projection de B sur l'axe horizontal $\cos\frac{4\pi}{3} = -\frac{1}{2} \times 3$
sur l'axe vertical $\sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2} \times 3$

Ex5 $b \leftrightarrow 3$; $e \leftrightarrow 2$ (ABC rectangle en A) $c \leftrightarrow 5$ $d \leftrightarrow 4$ $a \leftrightarrow 1$

Ex7 $\vec{AB}\begin{pmatrix}-4\\4\end{pmatrix}\cdot\vec{AC}\begin{pmatrix}-6\\-2\end{pmatrix} = 24 - 8 = 16$
$\vec{AB}\cdot\vec{AC} = AB \times AC \times \cos\widehat{BAC}$

$AB \times AC \times \cos\widehat{BAC} = 16$
donc $\cos\widehat{BAC} = \frac{16}{\sqrt{2\times16}\times\sqrt{8\times5}} = \frac{16}{16\sqrt{5}} = \frac{1}{\sqrt{5}}$

Ex8 $(\vec{AB}+\vec{AH})\cdot\vec{AB} = \vec{AB}\cdot\vec{AB} + \vec{AH}\cdot\vec{AB}$
$= 4 + AH^2 = 4 + (4-1) = 7$

$(\vec{AH}+\vec{HC})\cdot\vec{AB} = \vec{AC}\cdot\vec{AB}$
$(\vec{AH}+\vec{HC})\cdot\vec{AB} = \vec{AH}\cdot\vec{AB} + \vec{HC}\cdot\vec{AB}$
$= \underbrace{\vec{AH}\cdot\vec{AB}}_{AH^2} + \vec{HC}\cdot(\vec{AH}+\vec{HB})$
$= AH^2 + 0 - HB \times HC$
$= 3 - 2 = 1$

$(\vec{AH}+\vec{HB})\cdot(\vec{AH}+\vec{HC})$
$= AH^2 + \underbrace{\vec{AH}\cdot\vec{HC}}_{0} + \underbrace{\vec{HB}\cdot\vec{AH}}_{0} + \vec{HB}\cdot\vec{HC}$
$= 3 + 0 + 0 - 2$
$= 1$

Ex9 a) $m = -0{,}8$ b) $m = 0$ ou $m = 1$ c) $m = 1$

Ex10 $\vec{BA}\begin{pmatrix}-3\\-1\end{pmatrix}\cdot\vec{BC}\begin{pmatrix}2\\-6\end{pmatrix} = \cancel{18+3=40}\ -6+6 = 0$ donc ABC est rectangle en B.

Ex11 1) $\vec{BC}\begin{pmatrix}4\\-3\end{pmatrix}$ et $BC = \|\vec{BC}\| = \sqrt{25} = 5$ $\vec{BA}\begin{pmatrix}2\\2\end{pmatrix}\cdot\vec{BC}\begin{pmatrix}4\\-3\end{pmatrix} = 2$

2) $\vec{BA}\cdot\vec{BC} = (\vec{BH}+\vec{HA})\cdot\vec{BC}$
$= \vec{BH}\cdot\vec{BC} + \underbrace{\vec{HA}\cdot\vec{BC}}_{0}$
$= \vec{BH}\cdot\vec{BC}$

c) $\vec{BA}\cdot\vec{BC} = \vec{BH}\cdot\vec{BC} = BH \times BC$
et $\vec{BA}\cdot\vec{BC} = 2$
d'où $BH \times BC = 2$ $BH = \frac{2}{5}$ et $\frac{HC}{4} = \frac{23}{5}$

2b $\begin{cases}\vec{BA}\cdot\vec{BC} \geq 0 \\ \vec{BA}\cdot\vec{BC} \leq BC^2\end{cases}$ donc le projeté de A sur [BC] est un point du segment [BC].

Ex13 $\|\vec{R}\| = \|\vec{F_1}+\vec{F_2}\| = \sqrt{F_1^2+F_2^2+2\vec{F_1}\cdot\vec{F_2}} = \sqrt{90000+40000+2\times300\times200\times\cos50} \approx 455$ N

$W = \vec{F}\cdot\vec{d} = 2000 \times 50 \times \cos 45 \approx 70711$ J

Ex 14 $\vec{OB}\cdot\vec{OC} = (\vec{OA}+\vec{AB})\cdot(\vec{OA}+\vec{AC}) = OA^2 + \underbrace{\vec{OA}\cdot\vec{AC}}_{0} + \underbrace{\vec{AB}\cdot\vec{OA}}_{0} + \vec{AB}\cdot\vec{AC}$

$= 900 + 15\times 20 = 1200$

et $\vec{OB}\cdot\vec{OC} = OB\times OC\cos\alpha$

$= \sqrt{1125}\times\sqrt{1300}\cos\alpha$

D'où $\cos\alpha = \frac{1200}{\sqrt{1125\times 1300}}$

$\cos\alpha \approx 0,99$

$\alpha \approx 7°$

Ex 15 d : $3x-y+5=0$ $\vec{m}\begin{pmatrix}3\\-1\end{pmatrix}$

$M(x;y)\in\Delta \iff \vec{AM}$ et $\vec{m}$ sont colinéaires $\iff x+3y-7=0$

Ex 16 a) $\vec{u_1}\begin{pmatrix}2\\1\end{pmatrix}\cdot\vec{u_2}\begin{pmatrix}-3\\6\end{pmatrix}=0$ donc $d_1\perp d_2$

b) $\vec{u_1}\begin{pmatrix}1\\2\end{pmatrix}\cdot\vec{u_2}\begin{pmatrix}2\\1\end{pmatrix}\neq 0$ c) $\vec{u_1}\begin{pmatrix}1\\1+\sqrt2\end{pmatrix}\cdot\vec{u_2}\begin{pmatrix}-1\\1-\sqrt2\end{pmatrix} = -1+(-1)\neq 0$

Ex 17 $M(x;y)\in\mathcal{C}(A;5) \iff (x-1)^2+(y+2)^2=25 \iff x^2+y^2-2x+4y-20=0$

$M(x;y)\in\mathcal{C}(A;AB) \iff (x+1)^2+(y-2)^2=20 \iff x^2+y^2+2x-4y-15=0$

$M(x;y)\in\mathcal{C}(A;4) \iff (x-1)^2+(y+4)^2=16 \iff x^2+y^2-2x+8y+1=0$

Ex 18 a) $x^2+y^2-x-3y-5=0 \iff (x-\frac12)^2+(y-\frac32)^2=\frac{15}{2}$

$M\in\mathcal{C}(A;r)$ avec $A(1/2;3/2)$ et $r=\sqrt{7,5}$

b) $(x-2)(x+5)+(y-1)(y-4)=0 \iff x^2+3x-10+y^2-5y+4=0$

$\iff (x+\frac32)^2+(y-\frac52)^2=29/2$

$M\in\mathcal{C}(A;r)$ avec $A(-3/2;5/2)$ et $r=\sqrt{14,5}$

$A(2;1)$ et $B(-5;4)$
$\vec{AM}\cdot\vec{BM}=0$
$\iff M$ est sur le cercle de diamètre $[AB]$.

c) $3x^2+3y^2-6x-9y-1=0 \iff x^2+y^2-2x-3y-1/3=0$

$\iff (x-1)^2+(y-\frac32)^2=43/12$

$M(x;y)\in\mathcal{C}(A;r)$ avec $A(1;3/2)$ et $r=\sqrt{43/12}$

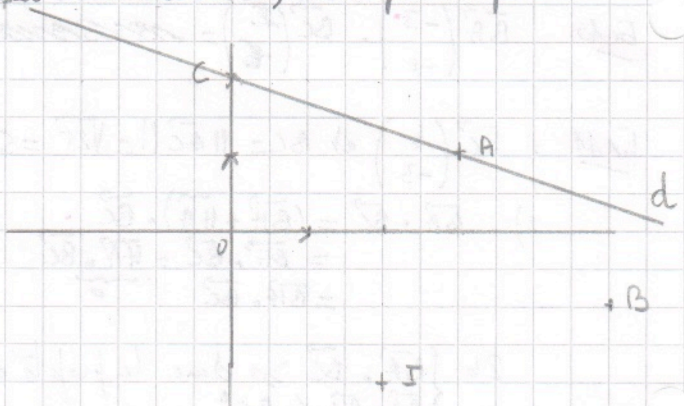
$\vec{AI}$ et $\vec{u}$ vecteur directeur de d sont-ils orthogonaux ? A est-il sur d ?

Ex 19 $\vec{AI}\begin{pmatrix}3\\-6\end{pmatrix}\cdot\vec{u}\begin{pmatrix}1\\1/2\end{pmatrix} = 3-3=0$ donc $\mathcal{C}$ et d sont tangents en A.

Ex 20 $\vec{OA}\begin{pmatrix}1\\2\end{pmatrix}\cdot\vec{u}\begin{pmatrix}1\\-1/2\end{pmatrix}=0$ où $\vec{u}$ est un vecteur directeur de d, droite passant par O.

$\mathcal{C}$ et d sont tangents en O.

Ex 21



$\mathcal{C}$: $x^2+y^2-4x+4y-2=0$

$(x-2)^2+(y+2)^2=10$

$\mathcal{C}$ a pour centre $I(2;-2)$ et rayon $\sqrt{10}$

d : $x+3y-6=0$ a pour vecteur directeur $\vec{u}\begin{pmatrix}-3\\1\end{pmatrix}$ et passe par $C(0;2)$

3a $\vec{IA}\begin{pmatrix}1\\3\end{pmatrix}$ et $\vec{u}\begin{pmatrix}-3\\1\end{pmatrix}$ sont-ils orthogonaux ?

$\vec{IA}\cdot\vec{u}=-3+3=0$ donc $\mathcal{C}$ et d tangents en A

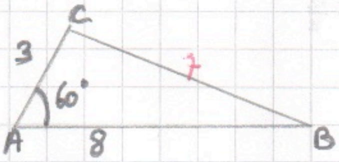
3b $M(x;y)\in d'$ vérifie $ax+by+c=0$

avec $\vec{IB}\begin{pmatrix}3\\1\end{pmatrix}\perp\vec{u'}\begin{pmatrix}-b\\a\end{pmatrix}$ $\vec{IB}\cdot\vec{u'}=0 \iff -3b+a=0$

$B(5;-1)\in d' \iff 5a-b+c=0$

Une équation de d' est $3x+y-14=0$

Ex 22



Al-Kashi $\quad BC^2 = AB^2 + AC^2 - 2\,AB \times AC \cos 60$

$BC^2 = 64 + 9 - 48 \times \frac{1}{2} = 49 \quad$ donc $BC = 7$

$AB^2 = AC^2 + BC^2 - 2\,AC \times BC \times \cos \widehat{ACB}$

$64 = 9 + 49 - 42 \cos \widehat{ACB}$

$\cos \widehat{ACB} = -1/7 \qquad \widehat{ACB} \approx 98°$

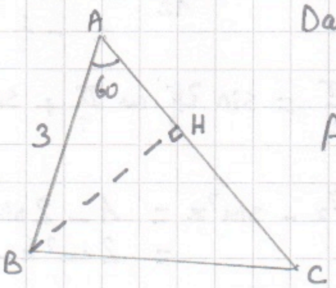
Ex 23

$BC^2 = AB^2 + AC^2 - 2\,AB \times AC \times \cos 60$

$= 9 + 16 - 24 \times 1/2$

$= 13$

Périmètre $= 7 + \sqrt{13}$



Dans ABH rectangle en H

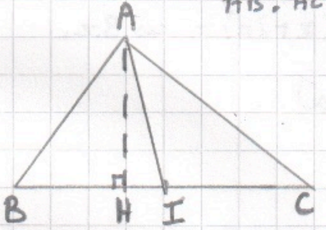
$\sin 60 = \frac{BH}{3}$ donc $BH = 3 \times \frac{\sqrt{3}}{2}$

Aire de $ABC = \frac{1}{2} \times 4 \times \frac{3\sqrt{3}}{2} = 3\sqrt{3}$

Ex 24 Théorème de la médiane

$AB^2 + AC^2 = \frac{1}{2} BC^2 + 2\,AI^2$

$\vec{AB} \cdot \vec{AC} = AI^2 - \frac{1}{4} BC^2$

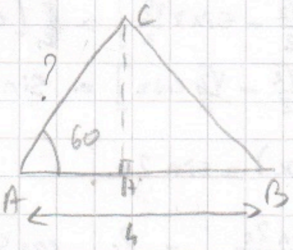


On a $64 + 81 = \frac{1}{2} \times 100 + 2\,AI^2$

d'où $AI^2 = \frac{95}{2}$ et $AI = \sqrt{\frac{95}{2}}$

Même calcul pour les deux autres médianes.

Ex 25



On a $\frac{1}{2} \times 4 \times CH = 5\sqrt{3}$ où H est la hauteur issue de C

donc $HC = \frac{5}{2}\sqrt{3}$

Dans AHC rectangle en H $\quad \sin 60 = \frac{HC}{AC}$

donc $AC = \frac{HC}{\sin 60}$

et $AC = \frac{5\sqrt{3} \times 2}{2 \times \sqrt{3}} = 5$

$BC^2 = AB^2 + AC^2 - 2\,AB \times AC \times \cos 60$

donc $BC^2 = 16 + 25 - 2 \times 4 \times 5 \times 0{,}5 = 21$ et $BC = \sqrt{21}$.

Ex 26

$\vec{AB} \cdot \vec{AC} = AB \times AC \times \cos(\widehat{BAC})$

donc $-12\sqrt{3} = 24 \cos(\widehat{BAC})$

et $\cos(\widehat{BAC}) = -\frac{\sqrt{3}}{2}$ donc $\widehat{BAC} = \frac{5\pi}{6}$ rad

Aire : cf n°23

Ex 27 Al-Kashi $\quad 7^2 = 3^2 + 8^2 - 2 \times 3 \times 8 \cos \widehat{BAC}$

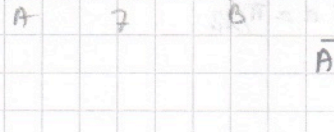
donc $\cos \widehat{BAC} = \frac{24}{48} = \frac{1}{2}$

Aire : cf n°23

$\sin^2 \widehat{BAC} + \cos^2 \widehat{BAC} = 1$

donc $\sin^2 \widehat{BAC} = 1 - \frac{1}{4} = \frac{3}{4}$ et $\sin \widehat{BAC} = \frac{\sqrt{3}}{2}$ car $\widehat{BAC} \in]0;\pi[$

Ex 28



$(\vec{AB} + \vec{AD})^2 = AB^2 + AD^2 + 2\,\vec{AB} \cdot \vec{AD}$

$\underbrace{\qquad}_{AC^2} = AB^2 + AD^2 + 2\,\vec{AB} \cdot \vec{AD}$

donc $\vec{AB} \cdot \vec{AD} = \frac{1}{2}(64 - 49 - 9) = 3$

$\vec{AB} \cdot \vec{AD} = AB \times AD \times \cos \widehat{BAD}$ et $\vec{AB} \cdot \vec{AD} = 3$

donc $\cos \widehat{BAD} = \frac{3}{7 \times 3} = \frac{1}{7}$

$\sin^2 \widehat{BAD} + \cos^2 \widehat{BAD} = 1$ et ~~sos~~ $\cos \widehat{BAD} = \frac{1}{7}$

donc $\sin^2 \widehat{BAD} = \frac{48}{49} = \frac{16 \times 3}{49}$ et $\widehat{BAD} \in]0;\pi[$

$\sin \widehat{BAD} = \frac{4}{7}\sqrt{3}$

Aire : cf n°23

Aire $ABCD = 2 \times$ Aire (ADC)

$= 2 \times$ Aire (ADB)

Ex 29 $\frac{5\pi}{12} = \frac{\pi}{6} + \frac{\pi}{4}$ donc $\cos\frac{5\pi}{12} = \cos\frac{\pi}{6} \times \cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4}$

$= \frac{\sqrt3}{2} \times \frac{\sqrt2}{2} - \frac{1}{2} \times \frac{\sqrt2}{2}$

$= \frac{1}{4}(\sqrt6 - \sqrt2)$

$\sin\frac{5\pi}{12} = \sin\frac{\pi}{6}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{\pi}{6}$

$= \frac12 \times \frac{\sqrt2}{2} + \frac{\sqrt2}{2} \times \frac{\sqrt3}{2} = \frac14(\sqrt2 + \sqrt6)$

$\frac{11\pi}{12} = \frac{2\pi}{3} + \frac{\pi}{4}$ donc $\cos\frac{11\pi}{12} = \cos\frac{2\pi}{3}\cos\frac{\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{\pi}{4}$

$= -\frac12 \times \frac{\sqrt2}{2} - \frac{\sqrt3}{2} \times \frac{\sqrt2}{2} = \frac14(-\sqrt2 - \sqrt6)$

$\sin\frac{11\pi}{12} = \sin\frac{2\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{2\pi}{3} = \frac{\sqrt3}{2} \times \frac{\sqrt2}{2} + \frac{\sqrt2}{2} \times \left(-\frac12\right) = \frac14(\sqrt6 - \sqrt2)$

Ex 30 $\cos(2x) = \cos^2x - \sin^2x = 1 - 2\sin^2x$

$= 2\cos^2x - 1$

a) $\cos 2x = 2\times\frac{3}{9} - 1 = -1/3$

b) $\cos 2x = 2\times\frac{9}{25} - 1 = -7/25$

c) $\cos 2x = 1 - 2\times\frac19 = \frac79$

Ex 31

$A(x) = \cos(7x)\sin(6x) - \sin(7x)\cos(6x)$

$= \sin(7x - 6x) = \sin x$

$B(x) = \cos x\cos 2x - \sin x\sin 2x$

$= \cos(x + 2x) = \cos 3x$

$C(x) = \cos 3x\sin 2x + \cos 2x\sin 3x = \sin(3x + 2x) = \sin(5x)$

Ex 32 $\sin\left(x - \frac{\pi}{3}\right) = \sin x\cos\frac{\pi}{3} - \cos x\sin\frac{\pi}{3} = \frac12\sin x - \frac{\sqrt3}{2}\cos x$

$\sqrt2\cos\left(x + \frac{\pi}{4}\right) = \sqrt2\left[\cos x\cos\frac{\pi}{4} - \sin x\sin\frac{\pi}{4}\right] = \sqrt2\left[\cos x \times \frac{\sqrt2}{2} - \sin x\frac{\sqrt2}{2}\right] = (\cos x - \sin x)$

$\sqrt2\sin\left(x - \frac{\pi}{4}\right) = \sqrt2\left[\sin x\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos x\right] = \sqrt2\left[\sin x\frac{\sqrt2}{2} - \frac{\sqrt2}{2}\cos x\right] = \sin x - \cos x$

Ex 33 $x \in \left]0; \frac{\pi}{2}\right[$. $\sin(3x)\cos x - \sin x\cos 3x = \sin(3x - x) = \sin 2x$

$\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x} = \frac{\sin 3x\cos x - \sin x\cos 3x}{\sin x \times \cos x} = \frac{\sin 2x}{\sin x\cos x} = 2$ car $\sin 2x = 2\sin x\cos x$

Ex 34 $a, b \in \left[0; \frac{\pi}{2}\right]$ $\cos a = \frac35$ donc $\sin^2 a = 1 - \frac{9}{25} = \frac{16}{25}$

$a \in \left[0; \frac{\pi}{2}\right]$ donc $\sin a \geq 0$ et $\sin a = \frac45$

$\cos^2 b = 1 - \frac14 = \frac34$

$b \in [0; \pi/2]$ donc $\cos b \geq 0$ et $\cos b = \frac{\sqrt3}{2}$

$\cos(a+b) = \cos a\cos b - \sin a\sin b = \frac35 \times \frac{\sqrt3}{2} - \frac45 \times \frac12 = \frac{1}{10}(3\sqrt3 - 4)$

$\sin(a+b) = \sin a\cos b + \sin b\cos a = \frac45 \times \frac{\sqrt3}{2} + \frac12 \times \frac35 = \frac{1}{10}(4\sqrt3 + 3)$

Ex 35 $\sin a = \frac12$ et $a \in \left[0; \frac{\pi}{2}\right]$ donc $\cos a = \frac{\sqrt3}{2}$

$\cos b = \frac14(\sqrt6 - \sqrt2)$ et $b \in \left[0; \frac{\pi}{2}\right]$ donc $\sin^2 b = 1 - \left(\frac{\sqrt6 - \sqrt2}{4}\right)^2 = \frac{1}{16}\left[16 - 6 - 2 + 4\sqrt3\right] = \frac{1}{16}\left[8 + 4\sqrt3\right]$

or $\left[\frac14(\sqrt6 + \sqrt2)\right]^2 = \frac{1}{16}(8 + 4\sqrt3)$ donc $\sin b = \frac{\sqrt6 + \sqrt2}{4}$ ($b \in [0; \pi/2]$ donc $\sin b \geq 0$)

Ex 36 $a \in \left[0; \frac{\pi}{2}\right]$ et $\cos a = \frac{\sqrt{2 + \sqrt3}}{2}$ $\cos 2a = 2\cos^2 a - 1 = 2 \times \frac14(2 + \sqrt3) - 1 = 1 + \frac{\sqrt3}{2} - 1 = \frac{\sqrt3}{2}$

$2a \in [0; \pi]$ et $\cos 2a = \frac{\sqrt3}{2}$ donc $2a = \frac{\pi}{6}$ d'où $a = \pi/12$

Ex 37 $\quad a \in \left]0; \frac{\pi}{4}\right[$

$$(\cos a + \sin a)^2 = \underbrace{\cos^2 a + \sin^2 a}_{} + \underbrace{2\cos a \sin a}_{}$$
$$= \quad 1 \quad + \quad \sin 2a$$

$$\frac{1+\sin 2a}{\cos 2a} = \frac{(\cos a + \sin a)^2}{\cos^2 a - \sin^2 a} = \frac{(\cos a + \sin a)(\cos a + \sin a)}{(\cos a - \sin a)(\cos a + \sin a)} = \frac{\cos a + \sin a}{\cos a - \sin a}$$

$$\frac{\cos\frac{\pi}{8} + \sin\frac{\pi}{8}}{\cos\frac{\pi}{8} - \sin\frac{\pi}{8}} = \frac{1+\sin\left(2\times\frac{\pi}{8}\right)}{\cos\left(2\times\frac{\pi}{8}\right)} = \frac{1+\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}} = \frac{1+\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}}\left(1+\frac{\sqrt{2}}{2}\right) = 1+\sqrt{2}$$

$$\frac{\cos\frac{\pi}{12} + \sin\frac{\pi}{12}}{\cos\frac{\pi}{12} - \sin\frac{\pi}{12}} = \frac{1+\sin\frac{\pi}{6}}{\cos\frac{\pi}{6}} = \frac{1+\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \sqrt{3}$$

Ex 38 $\quad$ 1_a $\quad$ Dans le triangle rectangle AOI, $\sin\frac{\hat{A}}{2} = \frac{4}{OA}$ et $\cos\frac{\hat{A}}{2} = \frac{8}{OA}$

1b
$$\sin^2\left(\frac{\hat{A}}{2}\right) + \cos^2\left(\frac{\hat{A}}{2}\right) = 1$$
$$\Leftrightarrow \frac{16}{OA^2} + \frac{64}{OA^2} = 1$$
$$\Leftrightarrow OA^2 = 80$$

$OA^2 = 80$ et $OA > 0$ donc $OA = 4\sqrt{5}$

$$\sin\left(\frac{\hat{A}}{2}\right) = \frac{4}{4\sqrt{5}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} \quad \text{et} \quad \cos\left(\frac{\hat{A}}{2}\right) = \frac{8}{4\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

On pose $\hat{u} = \frac{\hat{A}}{2}$ $\quad \sin(2\hat{u}) = 2\sin\hat{u}\cos\hat{u}$

d'où $\sin\hat{A} = 2\sin\frac{\hat{A}}{2}\cos\frac{\hat{A}}{2} = 2\times\frac{\sqrt{5}}{5}\times\frac{2\sqrt{5}}{5} = \frac{4\times5}{5\times5} = 0{,}8$

et $\cos^2\hat{A} = 1-\sin^2 A = 1-\left(\frac{4}{5}\right)^2 = \frac{9}{25}$ $\qquad \cos\hat{A} = \frac{3}{5} = 0{,}6$

2 $\quad$ Dans le triangle rectangle OBI, $\sin\frac{\hat{B}}{2} = \frac{4}{OB}$ et $\cos\frac{\hat{B}}{2} = \frac{6}{OB}$

$\sin^2\frac{\hat{B}}{2} + \cos^2\frac{\hat{B}}{2} = \frac{16+36}{OB^2}$ et $\sin^2 + \cos^2 = 1$ donc $OB^2 = 52$ $\qquad OB = \sqrt{52} = 2\sqrt{13}$

$$\sin\frac{\hat{B}}{2} = \frac{4}{\sqrt{52}} = \frac{2}{\sqrt{13}} \quad \text{et} \quad \cos\frac{\hat{B}}{2} = \frac{6}{\sqrt{52}} = \frac{3}{\sqrt{13}}$$

$$\sin\hat{B} = 2\sin\frac{\hat{B}}{2}\cos\frac{\hat{B}}{2} = 2\times\frac{2}{\sqrt{13}}\times\frac{3}{\sqrt{13}} = \frac{12}{13}$$

et $\cos^2\hat{B} = 1 - \frac{144}{13^2} = + \frac{25}{169}$ et $\cos\hat{B} = \frac{5}{13}$

3 $\quad \cos\hat{C} = \cos(\pi - \hat{A} - \hat{B}) = \cos(\pi - (\hat{A}+\hat{B})) = \cos\pi\cos(\hat{A}+\hat{B}) + \sin\pi\sin(\hat{A}+\hat{B})$
$$= -\cos(\hat{A}+\hat{B})$$

$\sin\hat{C} = \sin(\pi - (\hat{A}+\hat{B})) = \sin\pi\cos(\hat{A}+\hat{B}) - \cos\pi\sin(\hat{A}+\hat{B})$
$$= +\sin(\hat{A}+\hat{B})$$

On a donc $\cos\hat{C} = -\cos(\hat{A}+\hat{B}) = -\left[\cos\hat{A}\cos\hat{B} - \sin\hat{A}\sin\hat{B}\right]$
$$= -\left[\frac{3}{5}\times\frac{5}{13} - \frac{4}{5}\times\frac{12}{13}\right] = \frac{33}{65}$$

$$\sin\hat{C} = \sin(\hat{A}+\hat{B}) = \sin\hat{A}\cos\hat{B} + \sin\hat{B}\cos\hat{A}$$
$$= \frac{4}{5}\times\frac{5}{13} + \frac{12}{13}\times\frac{3}{5} = \frac{56}{65}$$